# Mathematics — Solved Paper 2020

## SECTION A (40 Marks)

*(Answer **all** questions from this Section)*

***Question* 1 :**

**(a) Solve the following Quadratic Equation :** **[3]**

$$x^2 - 7x + 3 = 0$$

**Give your answer correct to two decimal places.**

***Solution :***

$$x^2 - 7x + 3 = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1,\ b = -7 \text{ and } c = 3$$

$$= \frac{7 \pm \sqrt{49 - 4 \times 1 \times 3}}{2 \times 1}$$

$$= \frac{7 \pm \sqrt{37}}{2}$$

$$= \frac{7 \pm 6{\cdot}083}{2}$$

$$= \frac{7 + 6{\cdot}083}{2}, \frac{7 - 6{\cdot}083}{2}$$

$$= \frac{13{\cdot}083}{2}, \frac{0{\cdot}917}{2}$$

$$= 6{\cdot}542,\ 0{\cdot}459$$

$$= \mathbf{6{\cdot}54,\ 0{\cdot}46}$$ **Ans.**

**(b) Given** $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$

**If $A^2 = 3I$, where I is the identity matrix of order 2, find $x$ and $y$.** **[3]**

***Solution :***

$$A^2 = 3I \quad \Rightarrow \quad \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}\begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2 + 3y & 3x + 9 \\ xy + 3y & 3y + 9 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow \quad 3x + 9 = 0 \quad \text{and} \quad 3y + 9 = 3$$

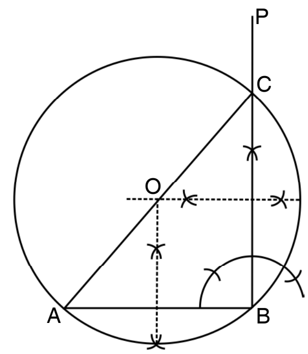
$$\Rightarrow \quad \mathbf{x = -3} \quad \text{and} \quad \mathbf{y = -2}$$ **Ans.**

**(c) Using ruler and compass construct a triangle ABC where AB = 3 cm, BC = 4 cm and ∠ABC = 90°. Hence construct a circle circumscribing the triangle ABC. Measure and write down the radius of the circle.** **[4]**

***Solution :***

**Steps :**

1. Draw AB = 3 cm.
2. Draw a line segment BP so that angle ABP = 90°.
3. From BP cut BC = 4 cm.
4. Join A and C to get the triangle ABC.
5. Draw perpendicular bisectors of AB and BC which intersect at point O on AC.
6. With O as centre and OA (= OC = OB) as radius draw the required circumcircle.




On measuring, we find the **radius of the circle = 2·5 cm.** **Ans.**

***Question* 2 :**

**(a) Use factor theorem to factorise $6x^3 + 17x^2 + 4x - 12$ completely.** **[3]**

***Solution :***

Substituting $x = -2$, we get

Remainder $= 6(-2)^3 + 17(-2)^2 + 4(-2) - 12$

$= -48 + 68 - 8 - 12 = 0$

$\therefore\ x = -2 \Rightarrow x + 2$ is a factor of the given expression.

$\mathbf{6x^3 + 17x^2 + 4x - 12} = (x + 2)\ (6x^2 + 5x - 6)$

$= (x + 2)\ (6x^2 + 9x - 4x - 6)$

$= (x + 2)\ [3x\ (2x + 3) - 2(2x + 3)]$

$= \mathbf{(x + 2)\ (2x + 3)\ (3x - 2)}$ **Ans.**

$6x^2 + 5x - 6$

$x + 2 \ )\ 6x^3 + 17x^2 + 4x - 12\ ($

$6x^3 + 12x^2$

$- \quad -$

$5x^2 + 4x - 12$

$5x^2 + 10x$

$- \quad -$

$- 6x - 12$

$- 6x - 12$

$+ \quad +$

×

**(b) Solve the following inequation and represent the solution set on the number line.**

$$\frac{3x}{5} + 2 < x + 4 \leq \frac{x}{2} + 5,\ x \in R$$ **[3]**

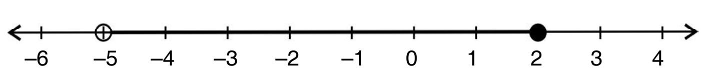
***Solution :***

$\Rightarrow\ 6x + 20 < 10x + 40 \leq 5x + 50$ [Multiplying by L.C.M. of 2 and 5 *i.e.* by 10]

$\Rightarrow\ 6x + 20 < 10x + 40$ and $10x + 40 \leq 5x + 50$

$\Rightarrow\ -4x < 20$ and $5x \leq 10$

$\Rightarrow\ x > -5$ and $x \leq 2$

$\Rightarrow$ 

**Ans.**

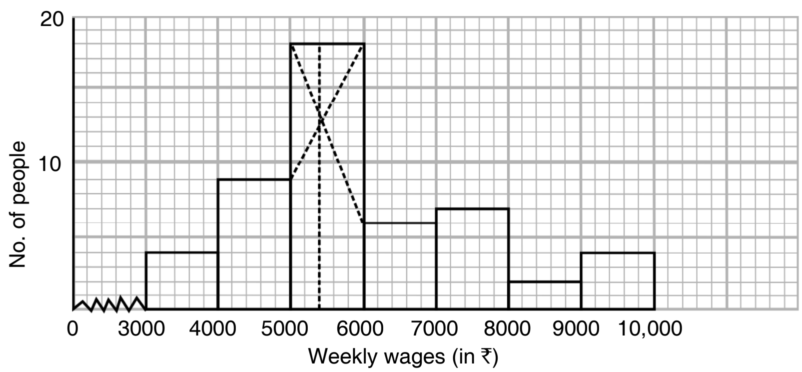
(c) **Draw a Histogram for the given data, using a graph paper :** **[4]**

| Weekly Wages (in ₹) | No. of People |
|---|---|
| 3000-4000 | 4 |
| 4000-5000 | 9 |
| 5000-6000 | 18 |
| 6000-7000 | 6 |
| 7000-8000 | 7 |
| 8000-9000 | 2 |
| 9000-10000 | 4 |

**Estimate the mode from the graph.**

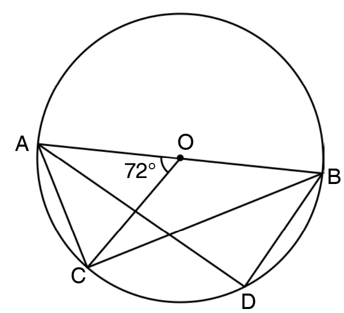
***Solution :***

Histogram for the given data will be as drawn below :




Clearly, **Mode** = **₹ 5,400 Ans.**

***Question* 3 :**




(a) **In the figure given alongside, O is the centre of the circle and AB is a diameter. If AC = BD and ∠AOC = 72°. Find :**

**(i) ∠ABC** **(ii) ∠BAD**

**(iii) ∠ABD** **[3]**

***Solution :***

(i) ∵ Angle at centre is twice the angle at remaining circumference

∴ ∠AOC = 2∠ABC

⇒ 72° = 2∠ABC

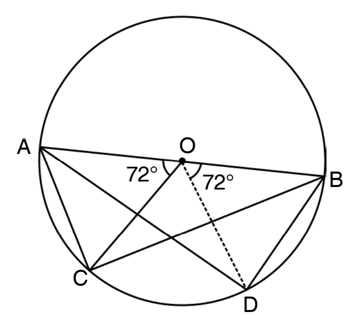
*i.e.* **∠ABC = 36°** **Ans.**

(ii) ∵ Equal chords subtend equal angles at centre

∴ ∠BOD = ∠AOC = 72°

⇒ 2∠BAD = 72° [∵ ∠BOD = ∠2BAD]

⇒ **∠BAD = 36°** **Ans.**




(iii) In ΔOBD, OB = OD (= radius)

⇒ $\angle OBD = \angle ODB = \dfrac{180° - 72°}{2} = 54°$

⇒ **∠ABD = 54°** **Ans.**

**(b) Prove that : $\dfrac{\sin A}{1+\cot A} - \dfrac{\cos A}{1+\tan A} = \sin A - \cos A.$** **[3]**

***Solution :***

$$\textbf{L.H.S.} = \frac{\sin A}{1+\dfrac{\cos A}{\sin A}} - \frac{\cos A}{1+\dfrac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\dfrac{\sin A+\cos A}{\sin A}} - \frac{\cos A}{\dfrac{\cos A+\sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\sin A+\cos A} - \frac{\cos^2 A}{\cos A+\sin A}$$

$$= \frac{\sin^2 A-\cos^2 A}{\sin A+\cos A}$$

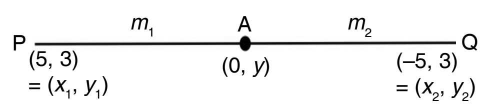
$$= \frac{(\sin A+\cos A)(\sin A-\cos A)}{\sin A+\cos A} = \sin A - \cos A = \textbf{R.H.S.}$$

**Hence proved.**

**(c) In what ratio is the line joining P(5, 3) and Q(–5, 3) divided by the *y*-axis ? Also find the coordinates of the point of intersection.** **[4]**

***Solution :***

Let the required point on $y$-axis be A(0, $y$).




Then $\quad 0 = \dfrac{m_1 \times -5 + m_2 \times 5}{m_1 + m_2}$

$\Rightarrow \quad 0 = -5m_1 + 5m_2$

$\Rightarrow \quad 5m_1 = 5m_2$ *i.e.* $\dfrac{m_1}{m_2} = \dfrac{5}{5} = \dfrac{1}{1} \Rightarrow \boldsymbol{m_1 : m_2 = 1 : 1}$

$m_1 : m_2 = 1 : 1 \Rightarrow$ A(0, y) is the mid-point of PQ

$\Rightarrow \quad y = \dfrac{3+3}{2} = 3$

$\therefore$ **The co-ordinates of the point of intersection** = (0, y) = **(0, 3)** **Ans.**

***Question* 4 :**

**(a) A solid spherical ball of radius 6 cm is melted and recast into 64 identical spherical marbles. Find the radius of each marble.** **[3]**

***Solution :***

Let the radius of each marble be $r$ cm

$\therefore$ Total volume of 64 marbles = Volume of solid spherical ball of radius 6 cm

$\Rightarrow \quad 64 \times \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi \times (6)^3$

$\Rightarrow \quad 64r^3 = 216$ *i.e.* $r^3 = \dfrac{216}{64} = \dfrac{27}{8}$

$\Rightarrow \quad \boldsymbol{r} = \dfrac{3}{2}$ cm = $\mathbf{1\dfrac{1}{2}}$ **cm** **Ans.**

**(b) Each of the letters of the word 'AUTHORIZES' is written on identical circular discs and put in a bag. They are well shuffled. If a disc is drawn at random from the bag, what is the probability that the letter is : [3]**

**(i) a vowel**

**(ii) one of the first 9 letters of the English alphabet which appears in the given word**

**(iii) one of the last 9 letters of the English alphabet which appears in the given word ?**

***Solution :***

Total number of outcomes for each part = 10

(i) Number of favourable cases = 5 [$a, u, o, i, e$]

$\therefore$ **Required probability** $= \frac{5}{10} = \mathbf{0{\cdot}5}$ **Ans.**

(ii) Favourable cases = $a, e, h$ and $i$

$\Rightarrow$ Number of favourable cases = 4

$\therefore$ **Required probability** $= \frac{4}{10} = \mathbf{0{\cdot}4}$ **Ans.**

(iii) Favourable cases = $u, t, r, z$ and $s$

$\Rightarrow$ Number of favourable cases = 5

$\therefore$ **Required probability** $= \frac{5}{10} = \mathbf{0{\cdot}5}$ **Ans.**

**(c) Mr. Bedi visits the market and buys the following articles : [4]**

**Medicines costing ₹ 950, GST @ 5%.**

**A pair of shoes costing ₹ 3,000, GST @ 18%.**

**A laptop bag costing ₹ 1,000 with a discount of 30%, GST @ 18%.**

**(i) Calculate the total amount of GST paid.**

**(ii) The total bill amount including GST paid by Mr. Bedi.**

***Solution :***

(i) **Total cost of total amount of GST**

= 5% of ₹ 950 + 18% of ₹ 3,000 + 18% of ₹ (1,000 – 30% of 1,000)

$= \frac{5}{100} \times$ ₹ 950 $+ \frac{18}{100} \times$ ₹ 3,000 $+ \frac{18}{100} \times$ ₹ 700

= ₹ 47.50 + ₹ 540 + ₹ 126 = **₹ 713.50** **Ans.**

(ii) **Total bill amount**

= Total cost + Total GST

= ₹ 950 + ₹ 3,000 + ₹ 700 + ₹ 713.50 = **₹ 5,363.50** **Ans.**

## SECTION B (40 Marks)

*(Answer any **four** questions from this Section)*

***Question* 5 :**

**(a) A Company with 500 shares of nominal value ₹ 120 declares an annual dividend of 15%. Calculate : [3]**

**(i) the total amount of dividend paid by the company**

**(ii) annual income of Mr. Sharma who holds 80 shares of the company.**

**If the return percent of Mr. Sharma from his shares is 10%, find the market value of each share.**

***Solution :***

(i) Dividend on 1 share = 15% of ₹ 120

= ₹ 18

$\Rightarrow$ **Total dividend paid** = Dividend on 500 shares

= ₹ 18 × 500 = **₹ 9,000** **Ans.**

(ii) **Annual income of Mr. Sharma** = Dividend on 80 shares

= ₹ 18 × 80 = **₹ 1,440** **Ans.**

10% of M.V. of each share = Dividend on 1 share

$\Rightarrow \frac{10}{100} \times$ M.V. = ₹ 18

$\Rightarrow$ **M.V. of each share** = ₹ 18 × $\frac{100}{10}$ = **₹ 180** **Ans.**

**(b) The mean of the following data is 16. Calculate the value of *f*.** **[3]**

| **Marks** | **5** | **10** | **15** | **20** | **25** |
|---|---|---|---|---|---|
| **No. of students** | **3** | **7** | ***f*** | **9** | **6** |

***Solution :***

| **Marks ($x$)** | **No. of students ($f$)** | **$f \times x$** |
|---|---|---|
| 5 | 3 | 15 |
| 10 | 7 | 70 |
| 15 | $f$ | $15f$ |
| 20 | 9 | 180 |
| 25 | 6 | 150 |
| | $\Sigma f = 25 + f$ | $\Sigma fx = 415 + 15f$ |

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\Rightarrow 16 = \frac{415 + 15f}{25 + f}$$

$$\Rightarrow 400 + 16f = 415 + 15f$$

$\Rightarrow f =$ **15** **Ans.**

**(c) The 4th, 6th and the last term of a geometric progression are 10, 40 and 640 respectively. If the common ratio is positive, find the first term, common ratio and the number of terms of the series.** **[4]**

***Solution :***

Let the first term = $a$, common ratio = $r$ and the number of terms of the series = $n$

4th term = 10 $\Rightarrow ar^3 = 10$,

6th term = 40 $\Rightarrow ar^5 = 40$ and

the last term = 640 $\Rightarrow ar^{n-1} = 640$

$\frac{ar^5}{ar^3} = \frac{40}{10} \Rightarrow r^2 = 4$ and $r = 2$ (given $r$ is positive)

$ar^3 = 10 \Rightarrow a \times (2)^3 = 10$ *i.e.* $a = \frac{10}{8} = \frac{5}{4}$

$ar^{n-1} = 640 \Rightarrow \frac{5}{4} \times (2)^{n-1} = 640$ *i.e.* $(2)^{n-1} = 640 \times \frac{4}{5} = 512 = 2^9$

$\Rightarrow n - 1 = 9$ and $n = 10$

$\therefore$ **The first term =** $\mathbf{\frac{5}{4}}$**, common ratio = 2**

and, **the number of terms in the series = 10** **Ans.**

***Question* 6 :**

**(a) If A =** $\begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$ **and B =** $\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$**. Find** $A^2 - 2AB + B^2$**.** **[3]**

***Solution :***

$\because \quad A^2 = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 9+0 & 0+0 \\ 15+5 & 0+1 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -12+0 & 6+0 \\ -20+1 & 10+0 \end{bmatrix} = \begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix}$

and, $B^2 = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 16+2 & -8+0 \\ -4+0 & 2+0 \end{bmatrix} = \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$

$\therefore \quad \mathbf{A^2 - 2AB + B^2} = \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - 2\begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - \begin{bmatrix} -24 & 12 \\ -38 & 20 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$

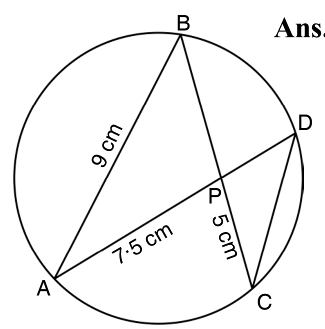
$= \begin{bmatrix} \mathbf{51} & \mathbf{-20} \\ \mathbf{54} & \mathbf{-17} \end{bmatrix}$ **Ans.**

**(b) In the given figure AB = 9 cm, PA = 7·5 cm and PC = 5 cm. Chords AD and BC intersect at P.**

**(i) Prove that ΔPAB ~ ΔPCD.**

**(ii) Find the length of CD.**

**(iii) Find area of ΔPAB : Area of ΔPCD.** **[3]**

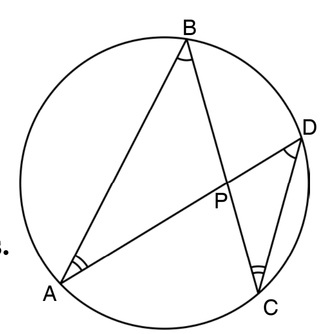



***Solution :***

(i) $\because$ Angles of the same segment are equal,

$\therefore \angle A = \angle C$ and $\angle B = \angle D$

$\Rightarrow$ **ΔPAB ~ ΔPCD** [By AA postulate]




(ii) $\because$ ΔPAB ~ ΔPCD

$\Rightarrow \frac{AB}{CD} = \frac{PA}{PC}$ *i.e.* $\frac{9}{CD} = \frac{7 \cdot 5}{5} \Rightarrow$ **CD = 6 cm Ans.**

(iii) $\because$ Ratio between the area of two similar triangles

= Ratio between the squares of their corresponding sides

$\Rightarrow$ **Area of ΔPAB : Area of ΔPCD** $= \dfrac{\text{Area } \Delta PAB}{\text{Area } \Delta PCD} = \dfrac{AB^2}{CD^2}$

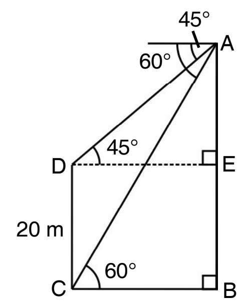
$$= \frac{9^2}{6^2} = \frac{9 \times 9}{6 \times 6} = \mathbf{9 : 4} \qquad \textbf{Ans.}$$

**(c) From the top of a cliff, the angle of depression of the top and bottom of a tower are observed to be 45° and 60° respectively. If the height of the tower is 20 m, find :**

**(i) the height of the cliff**

**(ii) the distance between the cliff and the tower. [4]**

***Solution :***




According to the given statement, the figure will be as shown alongside.

Clearly, AB = cliff, CD = tower = 20 m and DE ⊥ AB.

(i) **To find the height of the cliff *i.e.* to find AB**

$\because \dfrac{AB}{BC} = \tan 60° \Rightarrow AB = BC\sqrt{3}$

and, $\dfrac{AE}{DE} = \tan 45° \Rightarrow AE = DE$ *i.e.* AE = BC

Given : DC = 20 ⇒ EB = 20 *i.e.* AB – AE = 20

$\Rightarrow BC\sqrt{3} - BC = 20$

$\Rightarrow BC(\sqrt{3} - 1) = 20$

and, $BC = \dfrac{20}{\sqrt{3}-1} = \dfrac{20}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{20(1{\cdot}732+1)}{3-1} = 27{\cdot}32$

∴ **Height of the cliff** = AB

$= BC\sqrt{3} = 27{\cdot}32 \times 1{\cdot}732$ m = **47·32 m** **Ans.**

(ii) **The distance between the cliff and the tower**

= BC = **27·32 m** **Ans.**

***Question*** **7 :**

**(a) Find the value of *p* if the lines, 5*x* – 3*y* + 2 = 0 and 6*x* – *py* + 7 = 0 are perpendicular to each other. Hence find the equation of a line passing through (–2, –1) and parallel to 6*x* – *py* + 7 = 0. [3]**

***Solution :***

$5x - 3y + 2 = 0 \Rightarrow 3y = 5x + 2 \Rightarrow y = \dfrac{5}{3}x + \dfrac{2}{3}$

*i.e.* slope of the line $= \dfrac{5}{3}$

$6x - py + 7 = 0 \Rightarrow y = \dfrac{6}{p}x + \dfrac{7}{p}$ *i.e.* its slope $= \dfrac{6}{p}$

$\because$ Lines are perpendicular to each other

$\therefore\ \frac{5}{3} \times \frac{6}{p} = -1 \Rightarrow$ **p = –10**

$\because\ 6x - py + 7 = 0$ has slope $= \frac{6}{p} = \frac{6}{-10} = -\frac{3}{5}$

**For the required line :** $m = -\frac{3}{5}$ and $(-2, -1) = (x_1, y_1)$

Equation of the line is :

$y - y_1 = m(x - x_1) \quad \Rightarrow y + 1 = -\frac{3}{5}(x + 2)$

*i.e.* $5y + 5 = -3x - 6 \Rightarrow \mathbf{3x + 5y + 11 = 0}$ **Ans.**

**(b) Using properties proportion find $x : y$, given :**

$$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}.$$ **[3]**

***Solution :***

Applying componendo, we get :

$$\frac{x^2 + 2x + 2x + 4}{2x + 4} = \frac{y^2 + 3y + 3y + 9}{3y + 9}$$

$$\Rightarrow \frac{(x+2)^2}{2(x+2)} = \frac{(y+3)^2}{3(y+3)} \quad i.e. \quad \frac{x+2}{2} = \frac{y+3}{3}$$

$$\Rightarrow 3x + 6 = 2y + 6 \quad i.e. \ 3x = 2y$$

$$\Rightarrow \frac{x}{y} = \frac{2}{3} \quad i.e. \ \mathbf{x : y = 2 : 3}$$ **Ans.**

***Alternative method :***

Applying dividendo, we get :

$$\frac{x^2 + 2x - 2x - 4}{2x + 4} = \frac{y^2 + 3y - 3y - 9}{3y + 9}$$

$$\Rightarrow \frac{x^2 - 4}{2x + 4} = \frac{y^2 - 9}{3y + 9} \quad i.e. \quad \frac{(x+2)(x-2)}{2(x+2)} = \frac{(y+3)(y-3)}{3(y+3)}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-3}{3} \quad i.e. \ 3x - 6 = 2y - 6$$

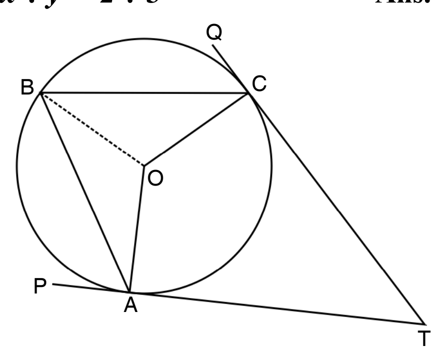
$$\Rightarrow 3x = 2y \quad i.e. \ \frac{x}{y} = \frac{2}{3} \text{ and } \mathbf{x : y = 2 : 3}$$ **Ans.**

**(c) In the given figure TP and TQ are two tangents to the circle with centre O, touching at A and C respectively. If ∠BCQ = 55° and ∠BAP = 60°, find :**

**(i) ∠OBA and ∠OBC**

**(ii) ∠AOC**

**(iii) ∠ATC.** **[4]**




***Solution :***

(i) $\because$ Angle between radius and tangent at the point of contact = 90°

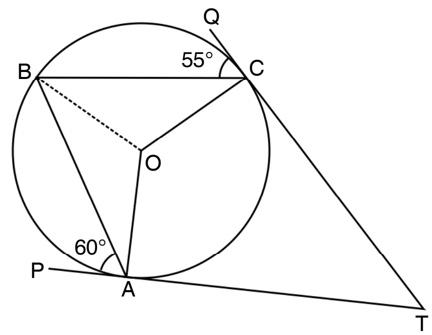
$\Rightarrow \quad \angle OAP = 90°$

$\angle OAB = \angle OAP - \angle BAP$

$= 90° - 60° = 30°$

$OB = OA$ (= radius)

$\Rightarrow \quad \mathbf{\angle OBA} = \angle OAB = \mathbf{30°}$




In the same way,

$\mathbf{\angle OBC} = \angle OCB$

$= 90° - 55° = \mathbf{35°}$ **Ans.**

(ii) $\angle AOC = 2\angle ABC$ [Angle at centre = 2 × Angle at remaining circumference]

$= 2(\angle OBA + \angle OBC)$

$= 2(30° + 35°)$

$= \mathbf{130°}$ **Ans.**

(iii) In quadrilateral AOCT :

$\angle ATC + \angle OAT + \angle OCT + \angle AOC = 360°$

$\Rightarrow \quad \angle ATC + 90° + 90° + 130° = 360°$

*i.e.* $\quad \mathbf{\angle ATC = 50°}$ **Ans.**

***Question* 8 :**

**(a) What must be added to the polynomial $2x^3 - 3x^2 - 8x$, so that it leaves a remainder 10 when divided by $2x + 1$ ? [3]**

***Solution :***

Let $p$ must be added

$\Rightarrow$ On dividing $2x^3 - 3x^2 - 8x + p$ by $2x + 1$, remainder is 10

$\Rightarrow 2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 - 8\left(-\frac{1}{2}\right) + p = 10$ $\quad [2x + 1 = 0 \Rightarrow x = -\frac{1}{2}]$

$\Rightarrow -\frac{2}{8} - \frac{3}{4} + 4 + p = 10$ *i.e.* $-\frac{1}{4} - \frac{3}{4} + 4 + p = 10$ and $\boldsymbol{p = 7}$ **Ans.**

**(b) Mr. Sonu has a recurring deposit account and deposits ₹ 750 per month for 2 years. If he gets ₹ 19,125 at the time of maturity, find the rate of interest. [3]**

***Solution :***

Given : $p$ = sum deposited per month = ₹ 750

$n$ = number of months = 2 × 12 = 24

and, maturity value (M.V.) = ₹ 19,125

$$P \times n + \frac{P \times n \times (n+1)}{2 \times 12} \times \frac{r}{100} = \text{M.V.}$$

$$\Rightarrow \quad 750 \times 24 + \frac{750 \times 24 \times 25}{2 \times 12} \times \frac{r}{100} = ₹\ 19{,}125$$

$$\Rightarrow \quad 18{,}000 + \frac{750r}{4} = ₹\ 19{,}125 \quad i.e. \quad \boldsymbol{r = 6\%}$$ **Ans.**

**(c) Use graph paper for this question. Take 1 cm = 1 unit on both *x* and *y* axes.**

**(i) Plot the following points on your graph sheets :**

**A(–4, 0), B(–3, 2), C(0, 4), D(4, 1) and E(7, 3)**

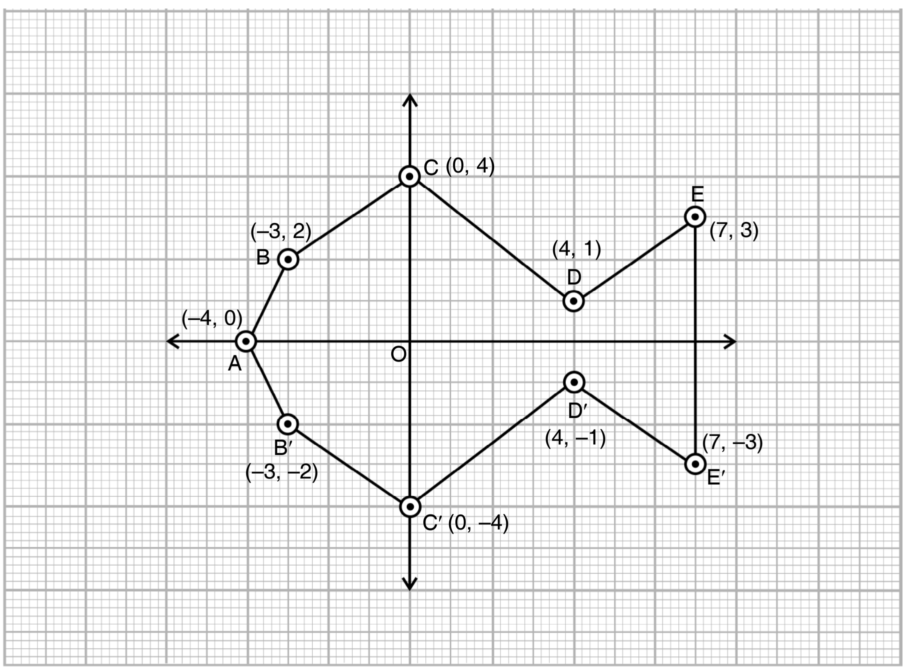
**(ii) Reflect the points B, C, D and E on the *x*-axis and name them as B′, C′, D′ and E′ respectively.**

**(iii) Join the points A, B, C, D, E, E′, D′, C′, B′ and A in order.**

**(iv) Name the closed figure formed.** **[4]**

***Solution :***

According to the given conditions, the figure formed will be of the form as given below :




(iv) The closed figure formed is a fish.

***Question* 9 :**

**(a) 40 students enter for a game of shot-put competition. The distance thrown (in metres) is recorded below :**

| **Distance in m** | **12-13** | **13-14** | **14-15** | **15-16** | **16-17** | **17-18** | **18-19** |
|---|---|---|---|---|---|---|---|
| **Number of students** | **3** | **9** | **12** | **9** | **4** | **2** | **1** |

**Use a graph paper to draw an ogive for the above distribution.**

**Use a scale of 2 cm = 1 m on one axis and 2 cm = 5 students on the other axis. Hence using your graph find :**

**(i) The median**

**(ii) Upper quartile**

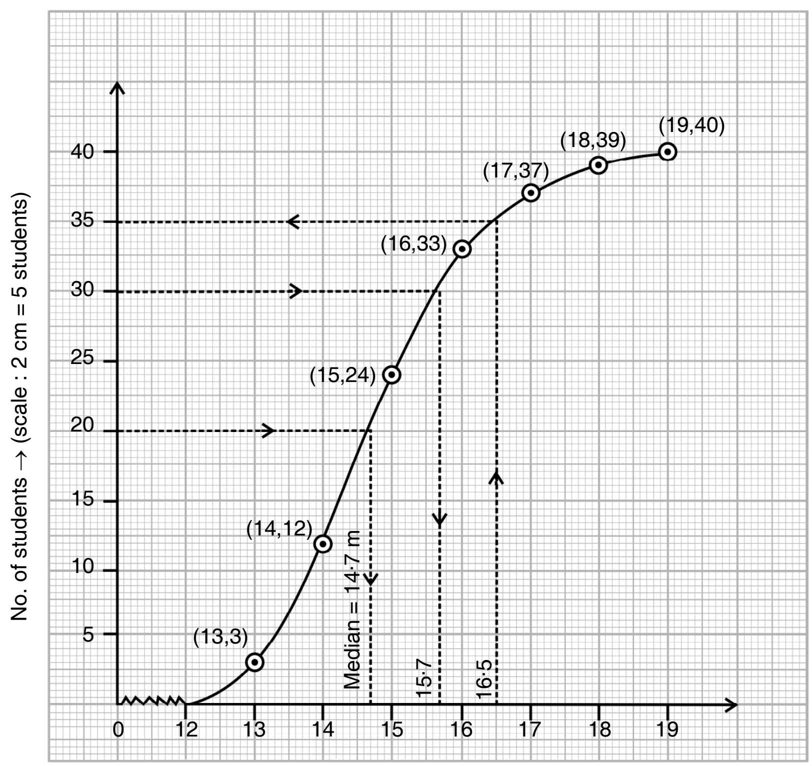
**(iii) Number of students who cover a distance which is above $16\frac{1}{2}$ m.** **[6]**

***Solution :***

| C.I | 12-13 | 13-14 | 14-15 | 15-16 | 16-17 | 17-18 | 18-19 |
|---|---|---|---|---|---|---|---|
| *f* | 3 | 9 | 12 | 9 | 4 | 2 | 1 |
| *c.f.* | 3 | 12 | 24 | 33 | 37 | 39 | 40 |

Plot the points (13, 3), (14, 12), (15, 24), (16, 33), (17, 37), (18, 39) and (19, 40) on a graph paper.

Draw a free-hand curve (ogive) passing through the points marked.




(i) **Median** = value of $\left(\frac{40}{2}\right)^{th}$ term = 20$^{th}$ term = **14·7 m** **Ans.**

(ii) **Upper quartile** = 40 × $\frac{3}{4}^{th}$ term = 30$^{th}$ term = **15·7 m** **Ans.**

(iii) **Number of students who cover a distance which is above $16\frac{1}{2}$ m (16·5 m)**

= 40 – 35 = **5** **Ans.**

**(b) If $x = \dfrac{\sqrt{2a+1} + \sqrt{2a-1}}{\sqrt{2a+1} - \sqrt{2a-1}}$, prove that $x^2 - 4ax + 1 = 0$.** **[4]**

***Solution :***

$$\Rightarrow \quad \frac{x}{1} = \frac{\sqrt{2a+1} + \sqrt{2a-1}}{\sqrt{2a+1} - \sqrt{2a-1}}$$

On applying componendo and dividendo, we get :

$$\frac{x+1}{x-1} = \frac{\sqrt{2a+1} + \sqrt{2a-1} + \sqrt{2a+1} - \sqrt{2a-1}}{\sqrt{2a+1} + \sqrt{2a-1} - \sqrt{2a+1} + \sqrt{2a-1}}$$

$$= \frac{2\sqrt{2a+1}}{2\sqrt{2a-1}} = \frac{\sqrt{2a+1}}{\sqrt{2a-1}}$$

Squaring both the sides, we get :

$$\frac{x^2+2x+1}{x^2-2x+1} = \frac{2a+1}{2a-1}$$

Again taking componendo and dividendo of both the sides, we get :

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1} = \frac{2a+1+2a-1}{2a+1-2a+1}$$

$\Rightarrow \quad \dfrac{2x^2+2}{4x} = \dfrac{4a}{2} \qquad$ *i.e.* $\quad 4x^2 + 4 = 16ax$

$\Rightarrow \quad x^2 + 1 = 4ax \qquad$ *i.e.* $\quad \mathbf{x^2 - 4ax + 1 = 0}$ **Ans.**

***Question* 10 :**

**(a) If the $6^{th}$ term of an A.P. is equal to four times its first term and the sum of first six terms is 75, find the first term and the common difference.** **[3]**

***Solution :***

Let the first term be $a$ and the common difference be $d$

Given : $6^{th}$ term of AP = 4 × first term $\Rightarrow a + 5d = 4a$

*i.e.* $3a = 5d$ ....I

Given : Sum of first six terms = 75

$\Rightarrow \quad \dfrac{6}{2}[2a + 5d] = 75$ and $2a + 5d = 25$ ....II

On solving equations I and II, we get :

$\mathbf{a = 5}$ **and** $\mathbf{d = 3}$ **Ans.**

**(b) The difference of two natural numbers is 7 and their product is 450. Find the numbers.** **[3]**

***Solution :***

Let the natural number be $x$ and $x + 7$

$\therefore \quad x(x + 7) = 450 \quad \Rightarrow \quad x^2 + 7x - 450 = 0$

$\Rightarrow \quad x^2 + 25x - 18x - 450 = 0$

*i.e.* $x(x + 25) - 18(x + 25) = 0 \Rightarrow (x + 25)(x - 18) = 0$

*i.e.* $x = -25$ or $x = 18 \Rightarrow x = 18$

$\therefore$ **Required natural numbers** = 18 and 18 + 7 = **18 and 25** **Ans.**

**(c) Use ruler and compass for this question. Construct a circle of radius 4·5 cm. Draw a chord AB = 6 cm.**

**(i) Find the locus of points equidistant from A and B. Mark the point where it meets the circle as D.**

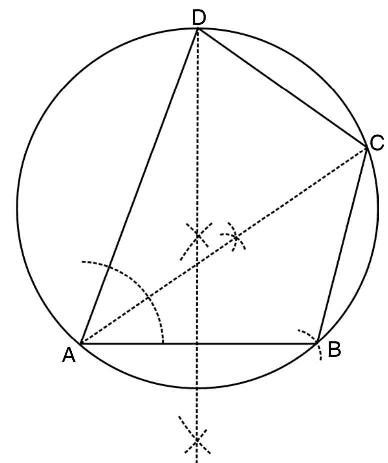
**(ii) Join AD and find the locus of points which are equidistant from AD and AB. Mark the point where it meets the circle as C.**

**(iii) Join BC and CD. Measure and write down the length of side CD of the quadrilateral ABCD.** **[4]**

***Solution :***

**Steps :**

1. Draw a circle with radius = 4·5 cm.
2. Draw a chord AB of length 6 cm.
3. Draw the perpendicular bisector of chord AB to get the locus of points equidistant from A and B which meets the circle at point D.
4. Join A and D.
5. Draw bisector of angle BAD to get the locus of points equidistant from AD and AB, which meets the circle at point C.
6. Get the quadrilateral ABCD and then measure CD.




**CD = 5 cm** (app.) **Ans.**

***Question* 11 :**

**(a) A model of a high rise building is made to a scale of 1 : 50.**

**(i) If the height of the model is 0·8 m, find the height of the actual building.**

**(ii) If the floor area of a flat in the building is 20 m$^2$, find the floor area of that in the model.** **[3]**

***Solution :***

Given scale factor $k = \frac{1}{50}$

(i) **Height of the model** = $k$ × Actual height of building

$\Rightarrow$ $0{\cdot}8 \text{ m} = \frac{1}{50} \times$ Actual height of building

$\Rightarrow$ **Actual height of building** = 0·8 m × 50 = **40 m** **Ans.**

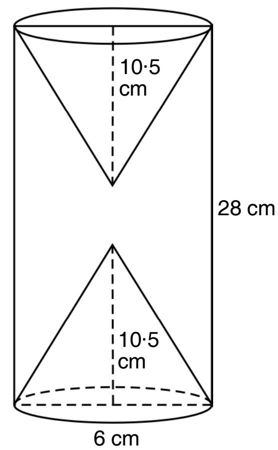
(ii) **Floor area of the model** = $k^2$ × Floor area of building

$= \left(\frac{1}{50}\right)^2 \times 20 \text{ m}^2 = \mathbf{0{\cdot}008 \text{ m}^2}$ **Ans.**

**(b) From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities are hollowed out. The diameters of the cones are also of 6 cm and height 10·5 cm.**

**Taking $\pi = \frac{22}{7}$ find the volume of the remaining solid.** **[3]**

***Solution :***




**For cylinder :**

Height H = 28 cm and radius $r = \frac{6}{2}$ cm = 3 cm

**For each cone :**

Height $h$ = 10·5 cm and radius $r$ = 3 cm

∴ **Volume of the remaining solid**

$$= \text{Volume of cylinder} - 2 \times \text{volume of cone}$$

$$= \pi r^2 H - 2 \times \frac{1}{3}\pi r^2 h$$

$$= \pi r^2 [H - \frac{2}{3} h]$$

$$= \frac{22}{7} \times (3)^2 \times [28 - \frac{2}{3} \times 10{\cdot}5] \text{ cm}^3$$

$$= \frac{22}{7} \times 9 \times 21 \text{ cm}^3 = \mathbf{594 \text{ cm}^3}$$ **Ans.**

**(c) Prove the identity $\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$.** **[4]**

***Solution :***

$$\textbf{L.H.S.} = \left(\frac{(1-\tan\theta)}{\left(1-\frac{1}{\tan\theta}\right)}\right)^2$$

$$= \frac{(1-\tan\theta)^2}{\left(\frac{\tan\theta - 1}{\tan\theta}\right)^2}$$

$$= \tan^2\theta \, \frac{(1-\tan\theta)^2}{(1-\tan\theta)^2}$$ $[\because (a-b)^2 = (b-a)^2]$

$= \tan^2\theta =$ **R.H.S.** **Hence proved.**

***Alternative method :***

$$\textbf{L.H.S.} = \frac{\left(1-\frac{\sin\theta}{\cos\theta}\right)^2}{\left(1-\frac{\cos\theta}{\sin\theta}\right)^2}$$

$$= \frac{\left(\frac{\cos\theta - \sin\theta}{\cos\theta}\right)^2}{\left(\frac{\sin\theta - \cos\theta}{\sin\theta}\right)^2}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \frac{(\cos\theta - \sin\theta)^2}{(\sin\theta - \cos\theta)^2}$$

$$= \tan^2\theta \times \frac{(\sin\theta - \cos\theta)^2}{(\sin\theta - \cos\theta)^2} \qquad [\because (a-b)^2 = (b-a)^2]$$

$= \mathbf{\tan^2\theta = R.H.S.}$ **Hence proved.**